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DESIGN OPTIMIZATION OF RC COLUMNS UNDER COMBINED AXIAL FORCE AND BENDING MOMENT USING EUROCODE 2 (EC2)

DIMENSIONNEMENT OPTIMAL DES POTEAUX EN BÉTON ARMÉ EN FLEXION COMPOSÉE SELON L'EUROCODE 2 (EC2)

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ABSTRACT

This work presents a method for optimizing the cost of reinforced ordinary concrete columns at ultimate limit state according to Eurocode2 (EC2). The objective function includes the costs of concrete, steel and formwork. All the constraints functions are set to meet design requirements of the European design Eurocode2 (EC2) and current practices rules. The optimization process is developed through the use of the Generalized Reduced Gradient algorithm. A typical design example is considered in order to illustrate the applicability of the proposed design model and solution methodology. It is concluded that this approach is economically more effective comparing to conventional design methods used by designers and engineers.

KEYWORDS: Design optimization, Reinforced Concrete columns, Eccentric compression, Eurocode2 (EC2), Algorithm.

RÉSUMÉ

Ce travail présente une méthode d'optimisation du coût des colonnes en béton ordinaire armé à l'état limite ultime selon l'Eurocode2 (EC2). La fonction "objectif" comprend les coûts du béton, de l'acier et du coffrage. Toutes les fonctions de contraintes sont définies pour répondre aux exigences de conception de l'Eurocode2 (EC2) et aux règles des pratiques actuelles. Le processus d'optimisation est développé à l'aide de l'algorithme du gradient réduit généralisé. Un exemple de conception typique est considéré afin d'illustrer l'applicabilité du modèle de conception et de la méthodologie de solutions proposées. Il est conclu que cette approche est économiquement plus efficace par rapport aux méthodes de conception conventionnelles utilisées par les concepteurs et les ingénieurs.

MOTS CLÉS: Conception optimale, Poteaux en béton armé, Compression excentrique, Eurocode2 (EC2), Algorithme.

1. INTRODUCTION

Columns are structural members in buildings carrying roof and floor loads to the foundations. Columns primarily carry axial loads, but most columns are subjected to moment as well as axial load. For each floor, the internal columns are designed for predominantly axial load while edge and corner columns are designed for axial load and bending moment simultaneously. The column section is generally square or rectangular, but circular and polygonal columns are used in special cases. When the section carries mainly axial load it is symmetrically reinforced. The symmetrical reinforcement is to be expected when the moment can change direction while keeping the same absolute value. Columns subjected to symmetric loading generally will be reinforced with a symmetric distribution of reinforcement. While symmetric reinforcement is often indicated for the interior columns of buildings, the minimum reinforcement solution is more economical for non-symmetric loading and finds many applications including the exterior columns of buildings for non-seismic regions. The traditional practice of providing symmetric reinforcement in column cross-section has been shown to be non-optimal for columns under non-symmetric design loads. An unsymmetrical reinforcement provides the most economical solution for the design of a column subjected to a small axial load and a large moment about one axis. The design of columns resisting moment and axial load is similar to the design in bending except that equilibrium should include bending moment and axial load.

Slender columns buckle and the additional moments caused by deflection must be taken into account by civil engineers and designers [1, 2, 3, 4, 5].

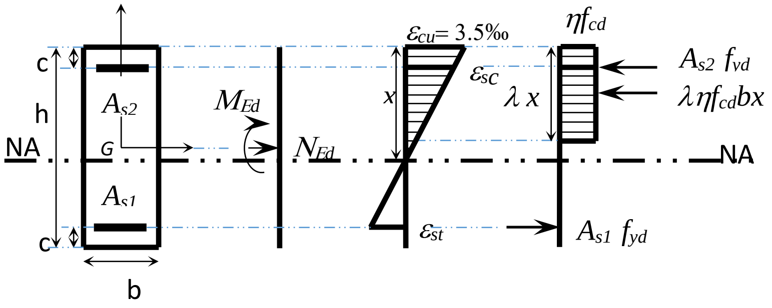
Economics and developments of methods of construction and advances in the methods of analysis of columns have all joined to shape the current practice. The economy of construction, design loads, required architectural design values, serviceability requirements, strength constraint and aesthetic design values or other requirements rules are all important for the choice of types of columns that are built. Safe and economical design of concrete structures can be achieved more by deciding on proper choices for the construction site, a practical overall layout of the structure and its resistant system, careful attention to construction detailing and sound construction practice. However, the overall cost reduction of the structure will strongly be dependent on a proper choice of construction materials including labor costs and on the optimal design of the individual structural elements as way be the case for large scale use in precast reinforced concrete component production. Because of the above mentioned advantages and the difficulty of the design, numerous journal articles have been published by researchers in recent years on the optimal cost design of reinforced concrete columns [6, 7, 8, 9, 10, 11, 12, 13, 14].

The search for an optimal design of RC columns that meets both performance and safety with minimum cost and lesser environmental impact was always the main goal pursued by most structural civil engineers and designers. This research presents a detailed objective function that considers the ratios cost not the absolute cost. It considers both shaping and material costs. Advances in numerical optimization methods, computer based numerical tools for analysis and design of structures and availability of powerful computing hardware have significantly helped the design process to ascertain the optimum design. The generalized reduced gradient (GRG) method is used to solve nonlinear programming problems. It is a very reliable and robust algorithm; also, various numerical methods have been used in engineering optimization [15, 16, 17, 18, 19, 20, 21].

This work presents a method for optimizing the cost of reinforced ordinary concrete columns at ultimate limit state according to Eurocode2 (EC2). The objective function includes the costs of concrete, steel and formwork. All the constraints functions are set to meet design requirements of the European design Eurocode 2 for RC columns and current practices rules. The optimization process is developed through the use of the Generalized Reduced Gradient algorithm. A typical design example is considered in order to illustrate the applicability of the proposed design model and solution methodology. It is concluded that this approach is economically more effective comparing to conventional design methods used by designers and engineers.

2. LIMIT STATE DESIGN OF REINFORCED CONCRETE COLUMNS

Design of columns is governed by the ultimate limit state; deflections and cracking during service conditions are not usually a problem, but nevertheless correct detailing of the reinforcement and adequate cover are important. For most columns, biaxial bending will not govern the design. The loading patterns necessary to cause biaxial bending in a building's internal and edge columns will not usually cause large moments in both directions. Corner columns way have to resist significant bending about both axes. This section is devoted to the design of a rectangular reinforced section at ultimate limit state solicited by flexural and axial loading (combined bending and axial compression) in accordance with provisions of Eurocode2 (EC2) design code[22].The assumptions used for the RC column cross section are respectively illustrated in Figure 1 (a), (b), (c), (d).



(a) Rectangular column section (b) External force (c) Strain (d) Stress block and internal force

Figure 1: (a) Rectangular column section, **(b)** External force, **(c)** Strain, **(d)** Stress block and internal force

Figure 1: (a) section rectangulaire du poteau **(b)** forces externes **(c)** diagramme de déformation **(d)** diagramme de contraintes et forces internes

In the present work, the rectangular stress block for concrete is used in order to obtain the optimal solutions solution. The elastic and perfectly plastic law for steel reinforcements is considered. In addition, the steel strain is considered unlimited in accordance with Eurocode2 (EC2) provisions. In Pivot B, the maximum strain capacity of upper fiber of concrete in compression is reached**.**

3. FORMULATION OF THE OPTIMIZATION PROBLEM FOR COLUMNS WITH UNSYMMETRICAL ARRANGEMENT REINFORCEMENT

3.1 Design variables

The design variables selected for the optimization are presented in Table 1.

Table 1 : Definition of design variables

Tableau 1 : Définition des variables de conception

Design variables	Defined variables
h	Height of cross section
b	Width of cross section
x	Depth of neutral axis from extreme compression fiber of concrete
A_{s1}	Area of tension reinforcement
A_{s2}	Area of compression reinforcement

3.2 Cost function

The objective function to be minimized in the optimization problems is the total cost of construction material of the column. This function can be defined as:

$$C_0 = C_c h.b + C_s(A_{s1} + A_{s2}) + 2C_f(h+b) \qquad (1)$$

Thus, the cost function to be minimized can be written as follows: $C=C_0/C_c$

$$C = \frac{C_0}{C_c} = h.b + \frac{C_s}{C_c}(A_{s1} + A_{s2}) + 2\frac{C_f}{C_c}(h+b) \qquad (2)$$

Where:

C_0	Total cost per unit height of column
C_s	Unit cost of reinforcing steel for the column
C_c	Unit cost of concrete for the column
C_f	Unit cost of formwork

3.3 Design constraints

a) Uniaxial bending constraint:

$$M_{Ed} \leq \lambda.\eta.f_{cd}.\ b.\ x.\ \left(\frac{h}{2} - c\right) + f_{yd}.A_{s2}.\left(\frac{h}{2} - \frac{\lambda x}{2}\right) - \ f_{yd}.A_{s1}.\left(\frac{h}{2} - c\right) \qquad (3)$$

(External moment, $M_{Ed} \leq$ Resisting moment of the cross section, M_{Rd})

b) Axial force equilibrium:

$$N_{Ed} = \lambda.\eta.f_{cd}.b.x + f_{yd}.(\mathrm{A}_{s2} - \mathrm{A}_{s1}) \qquad (4)$$

(Axial force equilibrium with rectangular stress block) (0.337

c) Constraint for minimum area of reinforcement

$$\frac{A_{s1}}{bh} \geq \ p_{\min} \qquad (5)$$

(Minimum steel percentage in tension)

$$\frac{A_{s2}}{bh} \geq p_{\min} \qquad (6)$$

(Minimum steel percentage in compression)

d) Constraint for maximum area of reinforcement:

$$\frac{A_{s1}+A_{s2}}{bh} \leq \ p_{\max} \qquad (7)$$

(Maximum steel percentage)

e) Conditions on strain compatibility in steel:

$$\frac{f_{yd}}{\gamma_s} \leq \varepsilon_{cu2}.\left(1 - \frac{c}{x}\right) \leq \varepsilon_{cu2} \qquad (8)$$

$$\frac{f_{yd}}{\gamma_s} \leq \varepsilon_{cu2}.\left(\frac{h}{x} - \frac{c}{x} - 1\right) \leq \infty \qquad (9)$$

Elasto-plastic behavior for steel and the pivot point is B, optimal use of steel requires that strains in steel must be limited to plastic region at the ultimate limit states (ULS)

f) Constraint of depth of neutral axis

$$c \leq x \leq h - c \qquad (10)$$

g) Constraints for maximum area of compression reinforcement:

$$f_{yd}.A_{s2}\,.\left(\frac{h}{2} - c\right) \leq 0{,}4\,.\ M_{Ed} \qquad (11)$$

h) Eccentricity constraint:

$$N_{Ed}\,.(h - 2c) - \left(M_{Ed} + N_{Ed}\left(\frac{h}{2} - c\right)\right) \leq (0337 + 0.81\frac{c}{h})\,.b.h^2.f_{cd} \qquad (12)$$

i) Non-slender: second order effects are ignored (limiting slenderness):

$$bh^3 \geq 0.103\frac{N_{Ed}l_0^2}{f_{cd}} \qquad (13)$$

(Column with the slenderness ratio ≤ the slenderness ratio limit)

j) Geometric design variable constraint including rules of current practice

$$1 \leq \frac{h}{b} \leq 4 \qquad (14)$$

$$h \leq \frac{l}{3} \qquad (15)$$

$$h_{\min} \leq h \leq h_{\max} \qquad (16)$$

$$b_{\min} \leq b \leq b_{\max} \qquad (17)$$

k) Non-negativity variables

$$h, b, A_{s1}, A_{s2}, x \geq 0 \qquad (18)$$

3.4 Optimization based on minimum cost design of reinforced concrete columns.

The optimum cost design of reinforced concrete columns under eccentric compression load can be stated as follows:
For given material properties, loading data and constant parameters, find the design variables defined in Table (1) that minimize the cost function defined in Eq. (2) subjected to the design constraints given in Eq. (3) through Eq. (18).

3.5 Solution methodology: Generalized Reduced Gradient method

The objective function Eq. (2) and the constraints equations, Eq. (3) through Eq. (18), together form a nonlinear optimization problem. The generalized reduced gradient (GRG) method is a well –known algorithm for the solution of the optimization problems with non-linear objective and constraints. Engineers and economists have used the GRG algorithm to solve industrial and economic problems. The GRG method has proven to be one of the more robust and efficient algorithm currently available for solving non-linear mathematical programming problems. The ideas for the Generalized Reduced Gradient algorithms were:
i) first formulated through the notion of constrained derivatives, ii) later it was developed using the name reduced gradient method, iii) and finally extended through the notion of generalized reduced gradient. The Non-Linear Programming (NLP) solution engine for Excel solver is GRG method: it is the optimizer employed by the solver optimization option within the spreadsheet packages Excel. The Generalized Reduced Gradient method is applied as it has the following advantages: j) The program can handle up to 200 constraints, which is suitable for reinforced ordinary columns design optimization problems, **jj)** GRG transforms inequality constraints into equality constraints by introducing slack variables. Hence all the constraints are of equality form. A more detailed description of the GRG method can be found in [23].

4. NUMERICAL RESULTS AND DISCUSSION

4.1 Design example

The column section resists an axial load of N_{Ed}=2.70 MN and a moment of M_{Ed}=0.80 MNm at ultimate limit state load pre-designed in accordance with provisions of Eurocode2 (EC2) design code. The pre-assigned parameters are defined as follows:
The corresponding pre-assigned parameters are defined as follows:
Clear height: l= 3.60m
Effective height: l_o= 0.7 l
Slenderness ratio: $\lambda_1 = l_o/i$
Radius of gyration about the axis considered: $i = (I/hb)^{0.5}$
Ultimate bending moment: $M_{Ed} = 1.35M_G + 1.5M_Q = 0.80$MNm
Ultimate axial force: $N_{Ed} = 1.35N_G + 1.5N_Q = 2.70$MN

Input data for concrete characteristics:
Strength class of concrete: C25/30.
Characteristic compressive cylinder strength of concrete at 28 days: f_{ck} = 25MPa.
Partial safety factor for concrete: γ_c = 1.5
Allowable compressive stress: f_{cd} = 16.67 MPa
$\lambda = 0.8$
$\eta = 1.00$
ε_{cu2}=3.5‰ (in the case of Pivot B, C25/30)
h_{min}=0.2 m
h_{max}=2 m
b_{min}=0.2 m
b_{max}=1 m
Input data for steel characteristics:
Steel class: S500
Yield strength: f_{yk} = 500MPa
Partial safety factor for steel: γ_s = 1.15
Allowable tensile stress: $f_{yd} = f_{yk}/\gamma_s$ = 435MPa
Young's modulus: E_s =2x105MPa
Yield strain, $\varepsilon_{yd} = f_{yd}/E_s = f_e/E_s.\gamma_s$, ε_{yd} =2.174‰ (steel HA Class S500)
Minimum steel percentage: p_{min} = 0.002
Maximum steel percentage: p_{max} = 0.04
Input data for unit's costs ratios of construction materials:
C_s/C_c = 10 and C_f/C_c = 0.01 for wood formwork

4.2 Comparison between the optimal cost design solution and the standard design approach

The vector of design variables, including the geometric dimensions of the column cross section, the area of tension reinforcement and the area of compression reinforcement as obtained from the standard design approach solution and the optimal cost design solution using the proposed approach, are shown in Table 2 below.

Table 2 : Optimal design solution for RC column

Tableau 2: Solution optimale pour poteau en béton armé

Design variables vector	Standard solution (Classical solution)	Optimal solution with minimum cost (S500, C25/30) C_s/C_c=10andC$_f$/C$_c$=0.01 wood formwork
h(m)	0.75	0.77
b(m)	0.40	0.33
$A_{s1}(m^2)$	0.000725	0.000511
$A_{s2}(m^2)$	0.001563	0.002136
x(m)	0.61	0.45
C	0.34588	0.30258
Gain	14%	

From the above results, it is clearly shown that a significant cost saving of the order of 14 % through the use of minimum cost design approach.

4.3 Parametric study

In this section, the optimal solution is obtained by considering (i) one of the dimensions of section is imposed (ii) the total area of reinforcement is imposed and (iii) imposed compressed reinforcing steel area.

Further practical requirements can also be implemented, such as symmetrical reinforcement, aesthetic, architectural and fire design values.

From the Table 3 below, it is clearly seen from the values of the relative costs C_s/C_c=10, C_f/C_c=0.01 and associated with the classical and optimal solutions corresponding for rectangular stress block, that the percentage saving is very important.

Table 3: Variation of relative gain with particular conditions imposed such as the section dimensions and reinforcing steel

Tableau 3 : Variation du gain relatif en fonction des conditions particulières imposées telles que la dimension de la section en béton et les sections des aciers

Optimal solution with imposed variable	Gain
b is imposed: b = 0.35m	10 %
$(A_{s1}+A_{s2})$ is imposed: $A_{s1}+A_{s2} \leq 0.0025m^2$	10 %
A_{s2} is imposed: $A_{s2} = 0.0017$ m^2	05 %

4.4 Cost sensitivity analysis

Comparing the relative gains can be determined for the various values of the unit cost ratios. The corresponding results are reported in Table 4 and Table 5 and illustrated graphically in Figure 2 and Figure 3.

The relative gains can be determined for various values of the unit cost ratios C_f/C_c for two given unit cost C_s/C_cratios. The corresponding results are reported in Table 4 and illustrated graphically in Figure 2 for C_s/C_c=10.

Table 4: Variation of relative gain in percent (%) versus unit cost ratio C_f/C_c of construction materials for C_s/C_c=10

Tableau 4: Variation du gain relatif en pourcentage (%) en fonction du rapport C_f/C_c des matériaux de construction pour C_s/C_c=10

	C_f/C_c	Gain(%)
C_s/C_c=10	0.01	14
	0.02	13
	0.03	12.50
	0.04	12
	0.05	11.60
	0.06	11
	0.07	10.90
	0.08	10.50
	0.09	10.30
	0.10	10

In order to further illustrate the variability of optimal solution with the unit cost ratio C_f/C_c,for a given value of C_s/C_c, the optimal solution has also been computed for various ratios C_f/C_c taken to be between 0.01 and 0.10.

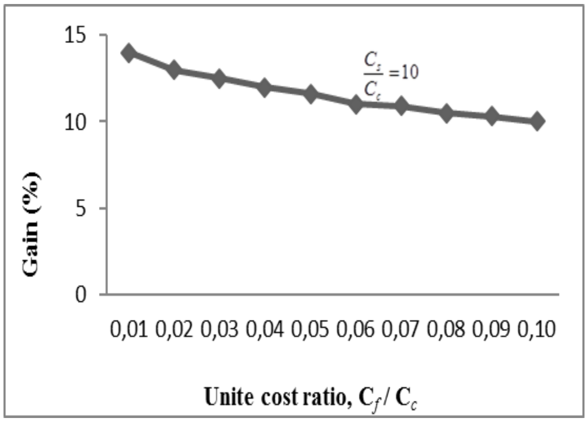


Figure 2 : Variation of relative gain in percentage for a given unit cost ratios, C_s/C_c=10 for different values of C_f/C_c

Figure 2 : Variation du gain relatif en pourcentage pour un rapport donné C_s/C_c=10 pour différents rapports C_f/C_c

From Table 4 and Figure 2, the gain decreases monotonically with the increasing of the unit cost ratio C_f/C_c.

The relative gains can be determined for various values of the unit cost ratios C_s/C_c for a given unit cost C_f/C_cratio. The corresponding results are reported in Table 5 and illustrated graphically in Figure 3 for C_f/C_c=0.01.

Table 5: Variation of relative gain in percent (%) versus unit cost ratio C_s/C_c of construction materials for C_f/C_c=0.01

Tableau 5: Variation du gain relatif en pourcentage (%) en fonction du rapport des coûts C_s/C_c des matériaux de construction pour C_f/C_c=0.01

C_f/C_c=0.01								
C_s/C_c	10	20	30	40	50	60	70	80
Gain(%)	14	11	10	08	07	05	04	03

In order to further illustrate the variability of optimal solution with the unit cost ratio C_s/C_c,for a given value of C_f/C_c, the optimal solution has also been computed for various ratios C_s/C_c taken to be between 10 and 80.

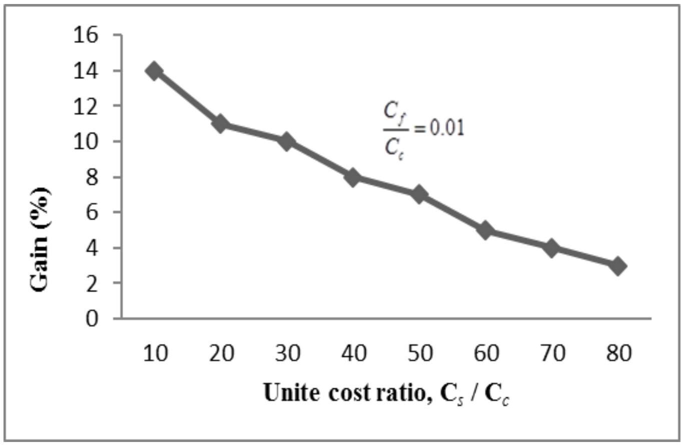


Figure 3 : Variation of relative gain in percentage for a given unit cost ratios, C_f/C_c=0.01 for different values of C_s/C_c

Figure 3 : Variation du gain relatif en pourcentage pour un rapport donné, C_f/C_c=0.01 pour différentes valeurs de C_s/C_c

From Table 5 and Figure 3, the gain decreases monotonically with the increasing of the unit cost ratio C_f/C_c.

5. CONCLUSIONS

The following important conclusions are drawn on the basis of this research:

i) The problem formulation of the optimal cost design of reinforced concrete columns can be cast into a nonlinear programming problem, the numerical solution is efficiently determined using the Generalized Reduced Gradient method in a space of only a few variables representing the concrete cross section dimensions.

ii) The optimal values of the design variables are only affected by the relative cost values of the objective function and not by the absolute cost values.

3i) The observations of the optimal solutions result reveal that the use of the optimization based on the optimum cost design concept may lead to substantial savings in the amount of the construction materials to be used in comparison to classical design solutions of reinforced concrete columns.

4i) The objective function and the constraints considered in this paper are illustrative in nature. This approach based on nonlinear mathematical programming can be easily extended to other sections commonly used in structural design. More sophisticated objectives and considerations can be readily accommodated by suitable modifications of the optimal cost design model. Analysis and design of RC columns in framed RC buildings under the standard fire, according to the Eurocode2 (EC2) can be developed without major alterations.

5i) In this work, the cost of formwork is included and it makes a significant contribution to the total costs.

6i) The suggested methodology for optimum cost design is effective and more economical comparing to the classical methods. The results of the analysis show that the optimization process presented herein is effective and its application appears feasible.

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APPENDIX

The following symbols are used in this paper:

AN	Neutral Axis
A_s	Area of tension reinforcement
A_s	Area of compression reinforcement
b	Width of section
c	Distance from the center of gravity of the reinforcement to the extreme fiber of concrete
C_0 /l	Total cost per unit height of column
C25/30	Class of ordinary concrete
C	Total relative cost of column
C_c	Unit cost of concrete for the column
C_f	Unit cost of formwork
C_s	Unit cost of reinforcing steel for the column
E_s	Young's elastic modulus of steel
f_{ck}	Characteristic compressive cylinder strength of ordinary at 28 days
f_{cd}	Design value of concrete compressive strength
f_{yk}	Characteristic elastic limit for steel reinforcement
G	Dead loads
h	Height of section
h_{max}	Maximum thickness of column
h_{min}	Minimum thickness of column
i	Radius of gyration about the axis considered
I	Second moment of area at the section about the axis
l	Clear height of column
l_o	Effective height of the column
M_{Ed}	Ultimate bending moment
M_G	Maximum design moments under

	dead loads
M_Q	Maximum design moments under live loads
N_{Ed}	Ultimate axial force
N_G	Maximum design force under dead loads
N_Q	Maximum design force under live loads
p_{min}	Minimum steel percentage
p_{max}	Maximum steel percentage
Q	Live loads
S500	Grade of steel
ε_{cu2}	Ultimate strain for the rectangular stress distribution compressive concrete design stress-strain relation
ε_{sc}	Strain of compressive reinforcement
ε_{st}	Strain of tensile reinforcement
ε_{yd}	Elastic limit strain
γ_c	Partial safety factor for concrete
γ_s	Partial safety factor for steel.
f_{yd}	Design yield strength of steel reinforcement
η	Design strength factor
λ	Compressive zone depth factor
λ_1	Slenderness ratio ($\lambda=l_o/i$)
λ_{1lim}	Slenderness limit