OPTIMIZATION OF REINFORCED CONCRETE SLABS ACCORDING TO EUROCODE 2-EC2

OPTIMISATION DES DALLES EN BÉTON ARMÉ SELON EUROCODE 2-EC2

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Abstract- This work presents a method for optimizing the cost of reinforced ordinary concrete and high strength concrete slabs at ultimate limit state according to Eurocode2 (EC-2). The objective function includes the costs of concrete, steel and formwork. All the constraints functions are set to meet design requirements of Eurocode2 and current practices rules. The optimization process is developed through the use of the Generalized Reduced Gradient algorithm. Two example problems are considered in order to illustrate the applicability of the proposed design model and solution methodology. It is concluded that this approach is economically more effective comparing to conventional design methods used by designers and engineers and can be extended to deal with other sections without major alterations.

Keywords: Optimization, High strength concrete, Reinforced concrete slabs, Eurocode2 (EC-2), Algorithm.

Résumé- Ce travail présente une méthode pour optimiser le coût du béton armé ordinaire et du béton à haute résistance des dalles à l'état limite ultime selon l'Eurocode2 (EC-2). La fonction ***objectif*** comprend les coûts du béton, de l'acier et du coffrage. Toutes les fonctions de contraintes sont définies pour répondre aux exigences de conception de l'Eurocode2 et aux règles de pratiques courantes. Le processus d'optimisation est développé grâce à l'utilisation de l'algorithme du Gradient Réduit Généralisé. Deux exemples sont considérés afin d'illustrer l'applicabilité du modèle de conception et de la méthodologie de la solution proposés. En conclusion, cette approche est économiquement plus efficace que les méthodes de conception classiques utilisées par les concepteurs et les ingénieurs et peut être étendue à d'autres sections sans altérations majeures.

Mots-clés : Optimisation, Béton à haute résistance, Dalles en béton armé, Eurocode2 (EC-2), Algorithme.

1. Introduction

Slabs are widespread structures, which constitute a very large part of the volume of all structures built for buildings and civil engineering. Slabs are the most vulnerable building elements and the most expensive to build and do not always offer the most economical solution. The remarkable advantages of solid reinforced concrete slabs that often justify its use and their mechanical performance. Slabs are used to prevent the building from twisting. The slab rigidifies the building on the horizontal plane, it makes the building steeper and less flexible. Because of the above mentioned advantages and the difficulty of the design, numerous journal articles have been published by researchers in recent years on the optimal cost design of reinforced concrete slabs [1, 2, 3, 4, 5].

High strength concrete slabs are frequently used in engineering practice. They are widely used for short to medium span highway bridges due to its moderate self weight, structural efficiency, ease of fabrication, fast construction and low maintenance. The majority of research studies conducted so far have focused on the optimization of ordinary concrete slabs whose strength class is between 12MPa and 50MPa, however only few studies have been done with regard to the optimization of high strength concrete slabs whose resistance class is between 50Mpa and 100MPa. The present optimization model is developed for ordinary and HSC slabs in flexure. Recent developments in the technology of materials have lead to the use of the high strength concrete. This is due mainly to its efficiency and economy. The reduction in the quantities of construction materials has enabled both a gain in weight reduction and in foundation's cost. HSC has a high compressive strength in the range of 50 to 100MPa; it has not only the advantage of reducing member size and story height but also the volume of concrete and the area of formwork. In terms of the amount of steel reinforcement, there is a substantial difference between the normal strength concrete structure compared to high strength concrete structures [6, 7].

In this work, it presents a detailed objective function that considers the ratios cost not the absolute cost. It considers both shaping and material costs. Advances in numerical optimization methods, computer based numerical tools for analysis and design of structures and availability of powerful computing hardware have significantly helped the design process to ascertain the optimum design. The generalized reduced gradient (GRG) method is used to solve nonlinear programming problems. It is a very reliable and robust algorithm; also, various numerical methods have been used in engineering optimization [8, 9, 10, 11, 12, 13, 14, 15, 16, 17].

This work presents a method for optimizing the cost of reinforced ordinary and high strength concrete slabs at ultimate limit state according to Eurocode2 (EC-2) [18]. The objective function includes the costs of concrete, steel and formwork. All the constraints functions are set to meet design requirements of Eurocode2-EC2 and current practices rules. The optimization process is developed through the use of the Generalized Reduced Gradient algorithm. Two example problems are considered in order to illustrate the applicability of the proposed design model and solution methodology. It is concluded that this approach is economically more effective comparing to conventional design methods used by designers and engineers and can be extended to deal with other sections without major alterations.

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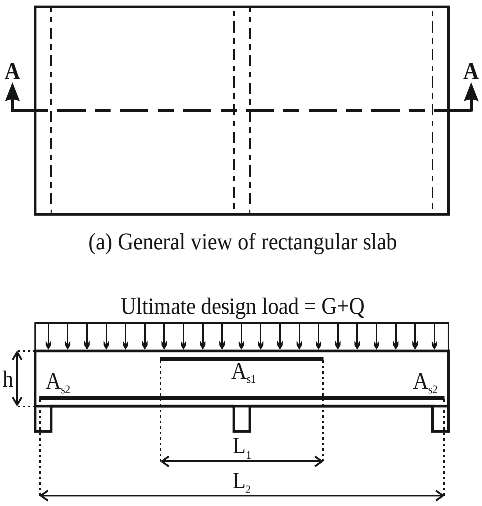
2. Ultimate limit state design of reinforced concrete slab under bending

The assumptions used for the typical reinforced slab cross section are respectively illustrated in Fig. 1(a), (b).
Consider the one-way continuous reinforced concrete slab shown in the **Fig. 1** with uniform thickness h and two equal spans.



(a) General view of rectangular slab

(b) Section **A-A**

Figure 1: Reinforced concrete slab: (a) General view, (b) Section A-A.

Figure 1: Dalle en béton armé: (a) Vue générale, (b) Section A-A.

The slab is assumed without loss of generality is only in one direction so that the study of the problem is reduced to that of a continuous beam with two equal spans (rectangular section: width b = 1.00m , effective depth d = 0.9h and h is the total thickness). The effect of shear is neglected. The rectangular stress block is used in this study (simplified stress bock for ultimate limit state). The bending-stress constraints must be considered for all n potentially critical sections. It should be noted that the bending moments are functions of the slab weight, they must be expressed in terms of the total thickness h.

3. Formulation of the optimization problem

3.1 Design variables

The design variables selected for the optimization are presented in Table 1.

Table 1: Definition of design variables

Tableau 1: Définition des variables de conception

Design variables	Defined variables
h	Thickness of slab
d	Effective depth
A_{si}	Area of reinforcing corresponding to the critical considered section (i=1,……,n)
α_i	Relative depth of compressive concrete zone

3.2 Cost function

The objective function to be minimized in the optimization problems is the total cost of construction material of the slab. This function can be defined as:

$$C_O = C_c Lhb + C_s \sum L_i A_{si} + C_f [L + 2b]h \quad \rightarrow \text{Minimum} \qquad (1)$$

Thus, the cost function to be minimized can be written as follows:

$$C = \frac{C_O}{C_c} = Lhb + \left(\frac{C_s}{C_c}\right) \sum L_i A_{si} + \left(\frac{C_f}{C_c}\right) [L + 2b]h \rightarrow \text{Minimum}$$

C_0 Total cost of slab
C_s Cost of unit steel area per unit area of slab
C_c Cost of concrete per unit volume
C_f Cost of unit formwork area per unit area of slab

The values of the cost ratios C_s/C_c and C_f/C_c vary from one country to another and may eventually vary from one region to another for certain countries [19, 20].

3.3 Design constraints

a)- Behavior constraints:

$$M_{Ed}(h) \leq \eta \lambda f_{cd} b_w d^2 \alpha_i (1 - 0{,}5\lambda\alpha_i) \qquad (3)$$

(External moment ≤Resisting moment of the cross section)

$$\alpha_i = \left(\frac{f_{yd}}{f_{cd}}\right)\left(\frac{A_{si}}{\eta\lambda d}\right) \quad (4)$$

(Internal force equilibrium with rectangular stress block)

$$\frac{A_{si}}{bd} \geq p_{min} \quad (5)$$

(Minimum steel percentage)

$$\frac{Asi}{bd} \leq p_{max} \quad (6)$$

(Maximum steel percentage)

Conditions on strain compatibility in steel:

$$\varepsilon_{cu3}\left(\left(\frac{1}{\alpha_i}\right) - 1\right) \geq \frac{f_{yd}}{E_s} \quad (7)$$

(Elasto-plastic behavior for steel and the pivot point is B, optimal use of steel requires that strains in steel must be limited to plastic region at the ultimate limit state (ULS))

$$\lambda\alpha_i(1-0{,}5\lambda\alpha_i) \leq \mu_{limit} \quad (8)$$

b)- Geometric design variable constraint

$$h_{min} \leq h \leq h_{max} \quad (9)$$

In accordance with EC-2, the possibility is offered to work with a rectangular stress distribution. This requires the introduction of a factor λ for the depth of the compression zone and a factor η for the design strength. The λ and η factors are both linearly dependent on the characteristic strength f_{ck} in accordance with the following equations [18]:

$$\lambda = 0.8 - \frac{f_{ck}-50}{400} \quad (10)$$

$$\mu = 1.0 - \frac{f_{ck}-50}{200} \quad (11)$$

With: $50 \leq f_{ck} \leq 90$MPa and λ=0.8, η =1.0 for $f_{ck} \leq 50$MPa

3.4. Optimization based on minimum cost design of reinforced concrete slabs

The optimum cost design of reinforced concrete slabs under ultimate limits state can be stated as follows:

For given material properties, loading data and constant parameters, find the design variables defined in Table (1) that minimize the cost function defined in Eq. (2) subjected to the design constraints given in Eq.(3) through Eq.(9).

3.5. Solution methodology: Generalized Reduced Gradient method

The objective function Eq. (2) and the constraints equations, Eq.(3) through Eq.(9), together form a nonlinear optimization problem. The reasons for the nonlinearity of this optimization problem are essentially due to the expressions of the cross sectional area, bending moment capacity and other constraints equations. Both the objective function and the constraint functions are nonlinear in terms of the design variables. In order to solve this nonlinear optimization problem, the generalized reduced gradient (GRG) algorithm is used. GRG nonlinear should be selected if any of the equations involving decision variables or constraints is nonlinear.

The Generalized Reduced Gradient method is applied as it has the following advantages: i) The GRG method is widely recognized as an efficient method for solving a relatively wide class of nonlinear optimization problems. ii) The program can handle up to 200 constraints, which is suitable for reinforced ordinary and HSC slabs design optimization problems. iii) GRG transforms inequality constraints into equality constraints by introducing slack variables. Hence all the constraints are of equality form.

4. Numerical results and discussion

4.1 Design example A for reinforced ordinary concrete slabs

The numerical example A corresponds to a concrete slab belonging to a pedestrian deck, with two continuous and equal spans pre-designed (initial design) in accordance with provisions of EC-2 design code.

The pre-assigned parameters are defined as follows:

L_1 =2m, $L=L_2$=5m + 5m;
Span length for the first and second span: l=5m
Live loads =Q=0.35 (t/m); dead loads=G=2.5 h (t/m), uniform distribution loading
$M_{Ed}(h)= 1.35M_G+ 1.5M_Q=0.05932h+0.00922$ MNm, critical section near the central region of the span $M_{Ed}(h)= 1.35M_G+ 1.5M_Q=0.10546h+0.01641$ MNm, critical section at the central support

Input data for ordinary concrete characteristics:
C20/25; f_{ck} = 20MPa; γ_c =1.5;f_{cd}= 11.33MPa; λ=0.80; η =1.00; ε_{c3}(‰)=3.5; h_{min}=0.15m; h_{max}= 0.40m

Input data for steel characteristics:
S400; f_{yk} = 400MPa; γ_s =1.15; $f_{yd}= f_{yk}/\gamma_s$ =348MPa
E_s =2x10^{5}MPa; $\rho_{min} = 0.26f_{ctm}/f_{yk}$=0.00143; ρ_{max} = 4%
f_{yd}/f_{cd}=30.71 for classes (S400, C20/25)
μ_{limit}= 0.392; for S400 and C20/25

Input data for units costs ratios of construction materials:

C_s / C_c = 30 for ordinary concrete
C_f / C_c = 0.01 for wood formwork

4.2 Comparison between the minimum cost design of ordinary concrete slabs

The optimal solutions using the minimum weight design is shown in Table 2 below.

Table 2: Comparison of the optimal solution with ordinary concrete to the initial design
Tableau 2 : Comparaison de la solution optimale à base de béton ordinaire avec la conception initiale

Design Variables Vector	Initial Design (Classical Solution)	Optimal solution with minimum cost (S400, C20/25) C_s/C_c=30, C_f/C_c=0.01 wood formwork
b(m)	1.00	1.00
h(m)	0.18	0.15
d(m)	0.16	0.135
$A_{S1}(m^2)$	6.9x10^{-4}	7.5x10^{-4}
$A_{S2}(m^2)$	4.2x10^{-4}	4x10^{-4}
α_1	0.166	0.213
α_2	0.10	0.115
Gain		18%

From the above results, it is clearly shown that a significant cost saving of the order of 18 % through the use of minimum cost design approach.

4.3 Design example B for reinforced high strength concrete slabs

The numerical example B corresponds to a high strength concrete slab belonging to a bridge deck, with two continuous equal spans pre-designed (initial design) in accordance with provisions of EC-2 design code.
The corresponding pre-assigned parameters are defined as follows:

L_1 =2m, $L=L_2$=5m+5m; span length for the first and second span: l=5m

Live loads =Q=0.45 (t/m); dead loads=G=2.5 h (t/m), uniform distribution loading
$M_{Ed}(h)= 1.35M_G+ 1.5M_Q=0.05932h+0.01185$ MNm, critical section near the central region of the span
$M_{Ed}(h)= 1.35M_G+ 1.5M_Q=0.10546h+0.0211$ MNm, critical section at the central support

Input data for HSC characteristics:
C70/85;f_{ck} = 70MPa; γ_c =1.5; f_{cd}= 46.67MPa; λ=0.75; η=0.90; ε_{cu3}(‰)=2.7; h_{min}=0.12m; h_{max}=0.30m

Input data for steel characteristics:
S500; f_{yk}=500MPa;γ_s=1.15; f_{yd}=f_{yk}/γ_s= 435MPa; E_s=2x10^5MPa; p_{min}=0.26f_{ctm}/f_{yk}=0.002392;

p_{max} = 4%.

μ_{limit}= 0.329; for S500 and C70/85

Input data for units costs ratios of construction materials:
C_s /C_c = 25 for high strength concrete
C_f /C_c = 0.10 for metal formwork

4.4 Comparison between the minimum cost design of high strength concrete slabs

The vector of design variables including the geometric dimensions of the slab cross section and the area of tension reinforcement as obtained from the standard design approach solution and the optimal cost design solution using the proposed approach, are shown in Table 3.

The optimal solutions using the minimum cost design is shown in Table 3 below.

Table 3: Comparison of the optimal solution with HSC to the initial design

Tableau 3: Comparaison de la solution optimale à base de béton à haute résistance avec la conception initiale

Design Variables Vector	Initial Design (Classical Solution)	Optimal solution with minimum cost (S500, C70/85) C_s/C_c=25, C_f/C_c=0.10 metal formwork
b(m)	1.00	1.00
h(m)	0.14	0.12
d(m)	0.126	0.108
$A_{S1}(m^2)$	6.8x10^{-4}	7.4x10^{-4}
$A_{S2}(m^2)$	3.7x10^{-4}	4.1x10^{-4}
α_1	0.07	0.09
α_2	0.04	0.05
Gain		14%

From the above results, it is clearly shown that a significant cost saving of the order of 14% through the use of minimum cost design approach.

5. Conclusions

The following important conclusions are drawn on the basis of this research:

- The problem formulation of the optimal cost design of reinforced concrete slabs can be cast into a nonlinear programming problem, the numerical solution is efficiently determined using the Generalized Reduced Gradient method in a space of only a few variables representing the concrete cross section dimensions.
- -The optimal values of the design variables are only affected by the relative cost values of the objective function and not by the absolute cost values.
- The observations of the optimal solutions results reveal that the use of the optimization based on the optimum cost design concept may lead to substantial savings in the amount of the construction materials to be used in comparison to classical design solutions of reinforced concrete slabs.
- The objective function and the constraints considered in this paper are illustrative in nature. This approach based on nonlinear mathematical programming can be easily extended to other sections commonly used in structural design. More sophisticated objectives and considerations can be readily accommodated by suitable modifications of the optimal cost design model.
- In this work, we have included the additional cost of formwork which makes a significant contribution to the total costs.
- The suggested methodology for optimum cost design is effective and more economical comparing to the classical methods. The results of the analysis show that the optimization process presented herein is effective and its application appears feasible.

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Appendix

The following symbols are used in this paper:

C20/25	Class of ordinary concrete
C70/85	Class of HSC
S400	Grade of steel
S500	Grade of steel
f_{ck}	Characteristic compressive cylinder strength of ordinary or HSC at 28 days
f_{cd}	Design value of concrete compressive strength
γ_c	Partial safety factor for concrete
η	Design strength factor
λ	Compressive zone depth factor
ε_{cu3}	Ultimate strain for the rectangular stress distribution compressive concrete design stress-strain relation
f_{yk}	Characteristic elastic limit for steel reinforcement
γ_s	Partial safety factor for steel.
f_{yd}	Design yield strength of steel reinforcement
ε_{yd}	Elastic limit strain
E_s	Young's elastic modulus of steel
p_{min}	Minimum steel percentage
p_{max}	Maximum steel percentage
α_i	Relative depth of compressive concrete zone
μ_{limit}	Limit value of reduced moment
l	Length of beam span
M_{Ed}	Ultimate bending moment
M_G	Maximum design moments under dead loads
M_Q	Maximum design moments under live loads
b	Width of slab (b=1.00m)
h	Total thickness of slab
d	Effective depth
A_{si}	Area of reinforcing steel
C_0	Total cost of slab
C_s	Cost of unit steel area per unit area of slab
C_c	Cost of concrete per unit volume
C_f	Cost of unit formwork area per unit area of slab